A New Model for the Cost of Equity of Unlisted Companies

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Abstract
A research carried out by FINLAB, Laboratory of Corporate Finance at the University of Cassino and Southern Lazio indeed revealed that analysts and consultants, for discounting the cash flows, used for non-listed companies, measures of the cost of equity calculated using the CAPM. This causes errors in the evaluations considered that the opportunity cost of capital thus calculated is undervalued respect to the risk. The model suggested in this paper, however, is reserved just to unlisted companies. Based on the assumption that in the absence of diversification the expected return must be commensurate with the specific risk as well as the systematic risk. The model consists of an algorithm that can estimate the cost of capital of an enterprise based on the multiple regression model, with a set of independent variables specially selected by the stepwise analysis.

Introduction
The valuation of investments, elementary and complex investments, which is the company, requires the choice of the discount rate able to approximate the cost to collect funds with the same characteristics of the flows to be discounted.

This consistency with the flows to be discounted is to conceive because of the risk associated with the transaction. In the case of risk-free investments will be necessary to employ a risk-free rate as opposed to risky investments where it will be necessary to apply a premium greater the higher the risk. Essentially the discounting under uncertainty conditions requires to the evaluator the adaptation of the rate to the risk profile of the flows. Being equity, the configuration that detects about it is not limited to the risk of insolvency but extends more broadly to the ability to realize a return lower than the expected level. Among the models used to estimate the capital cost the Capital Asset Pricing Model (CAPM) continues to be the one that receives greater consensus among researchers. This is a model of balance of financial markets, designed to calculate the equilibrium price of a financial asset, based on an expected return calculated as a linear function of the risk-free return and systematic risk activity that is multiplied by the PRM the risk premium of the market.

All this on the assumption that the investor by diversifying across multiple titles manage to cancel the specific risk. Except that diversification opportunities for investor in the venture capital of unlisted companies are much lower than those available for shareholders of listed companies. Normally the owner of an unlisted company and even more of small and medium ones because, engages in the company entirely his assets he assumes the total risk. Not being able to diversify and cancel the idiosyncratic risk he, in fact, is exposed to global enterprise risk, and not only to market risk. On the other hand the application of CAPM in listed companies is limited by the inability to calculate the beta made on the basis of market observations.

The literature
The cost of equity has its own complexity of estimation due to the nature of opportunity cost. The company, in fact, must ensure to the property the return that would have been achieved in other assets with the same risk profile. If this condition is not fulfilled, since the yield generated is lower than that achieved on other investments, the property would have no reason to continue funding the company and would address elsewhere the resources available.

Everything rests on the known relationship risk/return whose origin dates back in some way to the Expected Utility theory of Von Neumann and Morgenstern[1], with the affirmation of a rational decision makers and predictable in its decisions. In economic activity the behavioral patterns would predefined on the basis of the principle that the utility of wealth is always positive, considering the greatest wealth has always preferred to lower ones. Considering, then, that the decisions made under uncertainty can produce different outcomes, each with its own probability of realization, the individual would choose between different options that the expected utility of the top level instead of the one with the highest expected value. In this way, the expected utility theory introduces for the first time the risk which element which together usefulness individuals consider where they are asked to decide.

The utility function finds immediate application in decision making processes faces to the choice between several investments, each of which is characterized by the probability distribution of the final value. In essence, the policy choice will be the expected utility maximization, given the two variables, the average yield (μ) and variance (σ²) which features investment.

The paradigm mean-variance is also applicable when the decision is not about the individual but rather
combinations of individual investments or investment portfolios. For each portfolio it is in fact associated with a combination of the two parameters, always with a view to optimizing the selection. It follows that there are portfolios better than others. Considering that the utility function is growing compared to the expected return and it is negative respect the variance, it will surely be discarded portfolios that for the same expected return have a higher variance or in other words, a lower expected return with the same variance. The remaining portfolios together form the efficient frontier as locus of portfolios, each of which is not dominated by others. On this basis the theory of Markowitz [2], comes to affirm the principle that in order to build a portfolio of financial assets efficiently you need to select a combination of securities that allows to minimize the risk and maximize the return, also considered asynchrony between titles. A parity of return, in fact, the risk of the portfolios consist of two titles it can be reduced by choosing the titles which have a negative correlation, until reaching the resetting of the risk, in the case of perfect negative correlation i.e with correlation index equal to 1. Hence the differential value of diversification as compared to the concentration of a single title, since while the yield of the portfolio diversified across multiple titles coincides with the average returns associated with individual stocks, his risk will be lower in average risk of its components. Markowitz's theory finds its application in the model of the Capital Asset Pricing Model (CAPM) with which Sharpe [3] establishes the relationship between expected return and risk of the title. The model, doing just the value of the diversification, introduces for the first time the distinction between specific risk and systematic risk, where the first to the second difference can be canceled through diversification.

In the theoretical model of balance, the CAPM has been criticized, mostly oriented to emphasize the limits of its applicability in the reality far from the perfect market place, placed on the to basis of the paradigm. Fama and French [4] analyzing the performance of beta compared to equity returns in the period 1963-90, show the failure of CAPM in empirical texts and, therefore, conclude that many applications of the model are considered invalid. Several authors see weakness in the diversification of the CAPM, proving that often in reality investors, although for different reasons, they tend to focus their investments on a few titles (Blume and Friend, [5]; Polkovnichenko, [6]; Kumar and Goetzmann [7] Calvet, Campbell, and Sodini [8]).

The lack of diversification projects investors towards risk configuration inclusive of both components, systematic and specific, and towards an expected return able to remunerate also unsystematic risk (Levy [9] Merton [10] Malkiel and Xu [11], Barberis Huang [12], Boyle, Garlappi, Uppal, and Wang [13], Ang, Hodrick, Xing, and Zhang [14]).

Finally, as regards specifically the small unlisted companies Vos [15] argues that the Capital Asset Pricing Model is not adequate to represent the risk-return relationship.

The new model for unlisted companies
The CAPM in its classic version, however, best known and applied by analysts and researchers, predicts that the expected return of a stock is a function of a single factor that is the expected return of the market portfolio

$$\bar{R}_j = f(\bar{R}_m)$$

where:

$\bar{R}_j$ the expected return of stock $j$;

$\bar{R}_m$ the expected return of market portfolio $m$.

The relationship is linear, with an intensity equal to the coefficient $\beta$ which measures the degree of exposure of the title $j$ to market risk and, as such, can be expressed as measuring the market risk with the variance.

$$\beta_j = \frac{Cov(\bar{R}_j, \bar{R}_m)}{\text{Var}(\bar{R}_m)}$$

On the other hand, in accordance with the principle risk / return, the return expected by the investor shall be higher than risk free return $\bar{R}_f$, i.e certain return, produced by risk-free securities.

It follows that the expected return is equal to the risk free return plus a premium of return associated with the risk
of the title, in turn equal to the yield premium for the risk of the market portfolio, weighted by the coefficient $\beta$, namely:

$$\bar{R}_j = R_f + \beta_j PRM$$

also

$$\bar{R}_j = R_f + \beta_j (\bar{R}_m - R_f)$$

The correlation between the bond yield and performance of the market portfolio influence, therefore, the expected return. When this reaches higher values than the variance of the portfolio, $\beta$ will be greater than the unity and will amplify the risk premium of the market. With these features the title will be defined aggressive, given the greater volatility than the market risk, the investor will have the expectation of a higher return. Conversely, if the correlation remains lower than the variance of the portfolio, $\beta$ will be less than unity and will attenuate the risk premium of the market. The title will be defined defensive, given the lower volatility compared to the market risk, the investor will require a lower return.

Practically beta is estimated by regressing the returns of the stock relative to the returns of an index that is sufficiently representative of the market portfolio. The choice of the time interval on which to base the regression will be a real trade-off between the need to extend as much as possible the period so as to have a series of observations which guarantees the statistical significance and the constraint of avoiding the consideration of dated returns and, therefore, far from the current condition of the share.

Some empirical evidence of CAPM have quickly shown that the prices of shares belonging to the same sector often tend to move in the same direction, so as to lead to believe that the performance of the shares is influenced by risk factors common to groups of shares.

Conditioned also by these empirical results, the researchers pointed out, in a more or less strong, the limits of the CAPM. Among these that related to its excessive dependence on the market portfolio, as the only relevant factor for estimating the risk premium associated with a title, has represented the precondition of Arbitrage Pricing Theory (APT) developed by Ross[16].

The theory, while accepting, as the CAPM, the distinction between systematic risk and specific risk, suggests a different measurement method of the component that is not diversifiable, with recourse to a set of specific factors able to influence systematically the yield, according to the following linear relationship:

$$\bar{R}_j = R_f + \beta_{jA} PRM_A + \beta_{jB} PRM_B + \ldots$$

Assuming then that on the market cannot exist arbitrage opportunities, according to APT model, the prices and the bond yields depend on their covariance with the systematic risk factors, while the specific risk of each security can be eliminated through portfolio diversification.

In other words, the expected return of a security shall be the risk free return plus the sum of the expected values of risk premium associated with each factor multiplied by the sensitivity of the title to each of these factors, represented by some macroeconomic variables, such as' trends in real GDP, changes in market interest rates and the inflation rate. In this way the risk of systematic market is measured relative to a series of macro-economic factors rather than to a single and indistinct factor, represented by the market portfolio. To this end it is necessary to estimate the beta and the risk premium of each factor, based on the analysis of historical data related to investments and associated factors.

The CAPM shows even more limits when necessary to estimate the expected return from the investment in the equity capital of an unlisted company. In this case, because the investor can not resort to the diversification, the expected return and, therefore, the cost of equity will also reward also the specific risk as well as the systematic risk.

In this perspective, the work suggest a model suggests to estimate the cost of equity of unlisted companies have anyway as a reference the CAPM that the expected return is calculated using the equation:

$$\bar{R}_j = R_f + \beta (R_m - R_f)$$

where

$\bar{R}_j$ the expected return from the investment in the equity capital of the company $j$;
the risk free return; 

\( \beta \) coefficient; 

\( (R_m - R_f) \) risk premium of the market;

The relationship is appropriately corrected and supplemented to take account of the specificities of unlisted companies. First, the market beta is replaced by beta total, given the need to apply a weighting to the market premium that is explanatory of the entire risk, including the idiosyncratic risk. Therefore, according to the solution proposed by Damodaran [17], in turn taken over by Campbell et al. [18], I reconstruct the beta associated with the entire risk, idiosyncratic and systematic, that an unlisted company faces, instead of only systematic risk. In particular, the market beta expresses the variation of the systematic risk respect market risk. 

The total beta, then, can be understood as the coefficient of linear relationship between the overall risk of the asset (\( \sigma_j \)) (and the corresponding market risk of the asset (\( \sigma_m \)). That is 

\[
\beta_{\text{tot}} = \frac{\sigma_j}{\sigma_m}
\]

Given the expression of beta market, namely 

\[
\beta_m = \frac{\rho_{jm}}{\sigma_m} \sigma_j
\]

It is possible affirm 

\[
\beta_m = \beta_{\text{tot}} \rho_{jm}
\]

whence 

\[
\beta_{\text{tot}} = \frac{\beta_m}{\rho_{jm}}
\]

In conclusion, the total is equal to the beta of the market divided by the beta coefficient of correlation between the title of the company \( j \) and the market. 

Once determined the total beta, the model predicts that it is applied to listed companies to obtain the expected return from an investor who not resorting to diversification is exposed to systematic risk and specific risk. 

The expected return so calculated will regress compared to the explanatory variables considered in the overall risk. 

Each variable will be selected by stepwise regression, particularly suitable in exploratory studies like this paper, that in the presence of multiple variables aim to estimate a multivariate model. Among the variables to submit to stepwise selection cannot miss the current ratio, net working capital to total assets ratio, operating leverage, financial leverage, the interest coverage and the coverage ratio. Its observations will be designed with series of 10 years, from 2005 to 2014. 

The current ratio, the ratio of current assets and current liabilities, is a liquidity ratio used to determine if the company is able to meet its short-term financing through short-term assets. The net working capital or net trade working capital as the sum of accounts receivable, inventories and trade payables. The net working capital is one of the versions of working capital, which can take two different configurations, namely that of Net Working capital, which measures the difference between current operations and current operating liabilities and the net working capital flows, which measures instead, a difference between current assets and liabilities outstanding. 

This margin expresses the net investment related to the operating cycle acquisition - processing - sale and is, therefore, the investment required by current operations net of loans generated by the same management by the postponement of outputs negotiated with suppliers. In particular, a positive value expresses the financial requirements generated by the credit management/commercial debt, while a negative value indicates the need of the company to finance its non-financial accounts receivable with non-financial forms of debt. 

The feature of the net working capital is that it acts as a "sponge": if the capital is high, the sponge
absorbs liquidity, ie it expands, because growing trade receivables not collected, in this case there isn’t inventory turnover or decreased trade payables. If working capital is low, however, the sponge releases liquidity, that narrows and if assume negative values, would mean that the debts to suppliers are higher inventories and receivables. The operating leverage presents the measure of operative risk. Businesses, as mentioned, are exposed to the changing environment in different ways depending on their ability to adapt, modify and adapt the cost structure.

The demand from the market and sales business does not remain constant but change over time, both upwards and downwards, depending on the preferences of consumers. The demand connotes, in fact, strong elements of instability, due to the variability of tastes, the continually changing needs and expectations of consumers, more and more oriented towards a new and specific product design. The customer has abandoned the traditional approach to the product, based on the degree of satisfaction generated by its materiality, to approach to the concept of aggregate product, ie a product designed, for the most part, in terms of the services that the same can guarantee.

As a result, demand has become more complex, since customers are not looking for products and services but solutions to their problems, because no longer enough to produce a good and place it on the market but must also offer the customer an integrated system of services that can interact and respond effectively to the complex needs. In part the enterprise can counteract this variability trying to stabilize the demand, using, for example, marketing actions that improve the loyalty to the brand of the company, will enhance the overall usefulness of the product and, thereby, allow to contain the strong signs of discontinuity from the market.

Often this is not enough to solve the instability transmitted from the outside, being also necessary to provide adequate operating flexibility, which enables to the production structure of the company to adapt to changing environmental conditions and to ensure a greater certainty of performance. Only a low degree of rigidity of the corporate structure, the company may, in fact, adapt to any changes in production levels of the baseline scenario and, in this way, mitigate the uncertainty of operating results. For this purpose, the company must contain the incidence of fixed costs, in the knowledge that with the increase of fixed costs also increases the rigidity and becomes difficult to modify the company's production structure.

With reference to a well-defined time horizon and a given range of scale production, fixed costs, typically represented by amortization, administrative expenses and general expenses, are, in fact, almost independent of their level of production volumes, as opposed to costs variables, which, by definition, are closely related to the amount of production. In a structure characterized by a high incidence of fixed costs, increases the operational risk of the company and its exposure to the vagaries of the external environment, since any unfavorable trend of the market, with the consequent decline in revenue, it means, surely in operating results below expectations. In other words, given the fixed costs, the percentage change in profits that follows to a variation in the quantity produced is greater than the percentage change in the volume of production itself. In contrast, in a structure with high incidence of variable costs, it decreases the risk of the business and its sensitivity to the dynamics of market, given the opportunity to hold, reducing costs, the negative effects that a fall in sales can generate the performance period. This enterprise's ability to adapt to the movements of the demand is measurable by means of the operating leverage which, as noted, defines the reactivity or elasticity of EBIT to changes in the level of operational activity.

The operating leverage is an important indicator of the degree of operational risk, ie the risk of incurring losses induced by contractions in sales. With a high operating leverage increases the risk, given that, in these conditions, the decreases, even in a modest way of revenues, end up producing significant reductions in EBIT, quickly pushing the company to the area of loss. The operating leverage exerts, therefore, an amplification effect of variability existing that comes from outside and is influenced by some specific aspects of the core business, such as the existence of barriers to entry, the concentration of supply sources and the gradual obsolescence of the assets, as factors, in given conditions, of instability of the operating performance.

The leverage measures the financial risk the risk depends on the degree of autonomy from the outside, because the company is not a machine designed and built to operate according to patterns and procedures rigidly and deterministically fixed, it has not behaviors and dynamics completely predictable, or it is not system functionally independent from the external environment, or even a closed auto-poitetic system, that is able to produce their own components regenerating continuously their organization, thanks to internal processes that make it independent from the context. The debt reduces the degree of autonomy and, with it, the overall business risk increases. The choices about the financial structure may amplify the vulnerability of the company that arises from operational management. If the leverage rises the possibility of adapting the firm structure to possible contraction on sales decreases. This is because the financial costs associated with indebtedness are rigid compared to the volume of sales.

For this reason, some authors give a definition of financial risk that is linked only to all the manifestations of risk related to debt capital.

Fanni [21], argues that if projects are financed with new debt capital the degree of deviation from the
mean of the possible production increases than if it would financed solely with equity. According to this point of view, financial risk coincides with the risk related to debt capital that is destined to projects. Similarly Solomon [22] asserts that a part of the global risk of business originates in the operational management, ie in all the factors that contribute to shape the quality of the flow of expected operating net profits. We have to relate some of these factors to the forecasts of the economic and general trends; other factors are related to the forecasts about regions and markets in which company operates both considering the sales and supplying; finally other factors are related to the speed and flexibility with which the company is able to reduce its cost of production when its revenues contract. In normal conditions, that are characterized by the presence of borrowed capital in the company, it results that the overall risk depends on both the operational risk and the financial risk.

Instead, when the entity is financed solely with equity, the overall risk coincide with the operational risk. This is clearly a theoretical situation, since in reality the financial needs of businesses is always covered by both debt capital and equity.

The Leverage is useful to verify the relationship between equity capital and debt capital. If it is 1, it means that the company is financed solely through equity capital, if it is above 2 it implies that the debt capital is higher than the equity, while if it is between 1 and 2 it means that the equity capital is higher than the debt capital. This last case is considered the "ideal" case, but certainly it does not represent a rule. The Balance of the financial structure should be analyzed in conjunction with many other variables.

The Interest coverage is the ratio between EBITDA and financial charges. Interest coverage ratios are not simply an indicator of solvency, but also an indicator of the borrowing capacity of the company. Low values of the index indicate that the company has the potential to acquire new debt, ceteris paribus. Interest coverage ratios represent the number of times that the operating income or operating cash flows cover net interest.

Listed companies will be chosen in order to have sufficiently representative groups, according to the company for which we have to calculate the cost of capital. So the universe of companies is preliminarily distributed over 54 distinct groups that are distinguished by sector, using the ATECO classification (tab.1). Listed companies will be chosen because of the need to have sufficiently representative groups according to the company to evaluate. To this end, the universe of business is preliminarily distributed over 54 distinct groups by sector.

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<tr>
<th>Sector</th>
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<tbody>
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<td>Agriculture</td>
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<td>Mining industry</td>
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<td>9</td>
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<tr>
<td>Food industry</td>
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<td>Clothing</td>
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<tr>
<td>Leather and footwear</td>
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<td>Wood</td>
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<td>Paper</td>
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<td>Plastics and rubber</td>
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<td>Glass and cement</td>
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<td>Metallurgical and steel</td>
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<td>Electronic and electrical</td>
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<td>Vehicles-ships-Planes</td>
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<td>Furniture</td>
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<td>Otherindustriesresidual</td>
<td>32</td>
<td>32</td>
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<tr>
<td>Machineryrepairs</td>
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Each group includes business considered to be sufficiently homogeneous as regards the risk induced by sector. For each group the model predicts at least 20 listed companies that, after having carefully analyzed the core business, it is believed to be explanatory of the entire universe of companies belonging to the sector. On the whole, therefore, the framework of the model spread over 1,080 listed companies. These companies will be selected among the listed companies in the European markets, with the exception of English ones. It therefore prefers a database not concentrated in a single market on the basis of the large number of the panel to build and considered that the need to select high level of comparability will eliminate a large number of companies. In essence, the alternative solution to concentrate the panel on one market, in which the reduced numerosity of listed companies would lead to force the choice by selecting also companies with reduced degree of comparability. At the same time the decision to scrap the UK market is due to the belief that the LSE is too specific to companies listed in it can be comparable companies with a company belonging to one of the other European countries. In terms of methodology, therefore, the company will comment on the result of pooling of cross-sectional data and time series, that the comments will be two-dimensional, as it will vary by company and by time period of detection.
Based on the observations recorded for selected variables considered as explanatory of global risk and thus the return expected by the investor, the model provides for the multiple regression analysis between average yields expected.

Assuming that between the dependent variable and the explanatory variables there is a linear relationship, the multiple regression model may take the following expression:

\[ Y_i = b_0 + b_1 X_{1,i} + b_2 X_{2,i} + \ldots + b_n X_{n,i} + \varepsilon_i \]

Where

- \( Y_i \) is the expected return;
- \( b_0 \) is the intercept regression line;
- \( b_1, b_2, \ldots, b_n \) are returned by the regression coefficients, which represent the inclination of \( Y \) respect the variables \( X_1, X_2, \ldots, X_n \). In particular, \( b_1 \) is the inclination of \( Y \) respect to the the variable \( X_1 \) holding constant the variables \( X_2, \ldots, X_n \) etc.;
- \( X_1, X_2, \ldots, X_n \) are the accounting variables;
- \( \varepsilon_i \) is the error in correspondence of observation \( i \).

A once defined, the function can be applied to the non-listed company of which we need to determine the cost of capital. Holding constant coefficient values, \( b_1, b_2, \ldots, b_n \) and not changing the value of the expected return \( Y_i \), they enter values of fiscal variables, \( X_1, X_2, \ldots, X_n \) referring to the non-listed company to be valued. In this way, the function will return a new value of the expected return and, therefore, the cost of equity capital of an unlisted company.

**Conclusion**

The model suggested in this paper addresses the need to provide for non-listed companies metrics of cost of equity separate from those of listed companies. The framework is able to price the risk and by specific sectoral classification adopted allows a regression function for each of the 54 business groups. In a forthcoming work will proceed to test the model and to refine it further, especially with the selection of variables more suitable to explain both components of risk, the systematic and specific.

**References**

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